

Review of viscoelastic models applied to cortical bone



Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Yann Blake

17334432

*4B1 Mechanics of Solids module supervised by **Professor Mark Ahearne***

Department of Mechanical, Manufacturing and Biomedical Engineering, School of Engineering

Trinity College Dublin

August 2021

Abstract

Bone is a composite of the biopolymer collagen type I and the bioceramic hydroxyapatite. Two main types of regions are found in bones: cortical - the most important for its mechanical properties - and trabecular. Viscoelasticity is an essential property of cortical bone. Three viscoelastic models selected in the existing literature are analysed and compared against each other in this report. Based on the chosen strain rate range chosen, different models may be suitable to fit the data of an experiment. For mechanical testing at a macroscopic scale, a Maxwell-Weichert seems more suitable and adapted to a wider range of strain rates. At a microscopic scale, the Burgers model described in this report also provided satisfactory results. More extended analysis on the models are provided in this report.

Contents

1	Introduction	2
2	Three viscoelastic models applied to cortical bone	2
2.1	<i>Dynamic Viscoelastic Response of Bone</i>	3
2.2	<i>A viscoelastic, viscoplastic model of cortical bone valid at low and high strain rates</i>	3
2.3	<i>Viscoelastic properties of human cortical bone tissue depend on gender and elastic modulus</i>	3
3	Tennyson et al.'s simplified Voigt model	4
4	Johnson et al.'s Maxwell— Weichert model	4
5	Ziheng et al.'s Burgers model	5
6	Comparison and evaluation of the models	6
7	Conclusion	7

1 Introduction

Bone is a composite of the biopolymer collagen type I and the bioceramic hydroxyapatite with smaller amounts of other compounds: water, calcium, phosphate and proteoglycans.[1] Two main types of regions are found in bones: cortical and trabecular. The cortical bone is the most compact part which consists of a dense outer surface layer protecting the internal cavity of the bone.[2] Eighty percent of the skeletal mass is of the cortical type.[3]

Research on bones as parts of the skeleton framework, as a rigid tissue and as a material is essential in order to understand the general functions of the organism and predict some age-related changes for example. Osteoporosis is one of these undesired changes which leads to the thinning of the cortical region and a reduction of the bone density. Since cortical bone is essential to the body structure and to the weight resistance due to great mechanical properties, any changes (osteoporosis, fracture, etc.) then become major issues.[4, 5] Before trying to avoid these changes or finding solutions to compensate them such as implants [6], one must investigate the microstructural mechanical properties of the cortical bone.

Viscoelastic materials exhibit a time delay in returning the material sample to the original shape which leads to energy loss. Viscoelasticity is an essential property of cortical Bone.[7] Evidences of this are its creep and relaxation behaviours. Some research also show that the rate-dependent failure behaviour of bone is related to its viscoelasticity. With the rate-dependent strength and fatigue strength dependence on frequency of bone, evidences of a greater susceptibility to crack growth in bone was found, and in fact explained with its viscoelasticity.[8] What is insightful for researchers here, is the prediction ability of fracture risks we can have when understanding the bone viscoelasticity and its correlated parameters. As emphasized in literature, determining these parameters is quite sensitive considering the variations of properties at a microstructural level.[9] It remains however quite accurate to consider bone as a homogeneous (at least transversely) material given that comparison of the mechanical properties of single osteons (0.30-mm-wide fundamental functional units of cortical bone) with macroscopic samples of bone demonstrate great similarities.[10]

In addition to understanding fractures, viscoelasticity of cortical bone is a parameter of interest for other phenomenon. Bone remodeling and its overall strength also appear to be correlated with the viscoelastic properties.[11, 9]

These viscoelastic properties are found to vary between male and female bone at various scale according to some studies with the commonly used technique of nanoindentation. Nanoindentation is a non-destructive nanomechanical test that provides mechanical properties from precise compliance measurements.[9, 8, 12, 13] Cortical bone viscoelasticity is also related to the collagen and mineral properties, the porosities, as well as the bone hierarchical tissue organisation. It is found to increase with vascular porosity, while it decreases with the degree of mineralization of the extravascular matrix.[14] The biopolymer collagen fibers are also viscoelastic themselves which provides an additional explanation to the viscoelasticity of bones.[1]

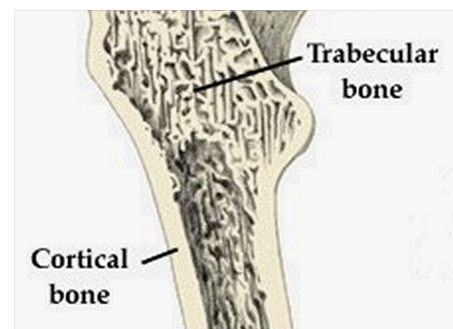


Figure 1: Cortical Bone vs Trabecular Bone [15]

2 Three viscoelastic models applied to cortical bone

A review and analysis of three research papers which propose different viscoelastic models, was carried out in this present report.

The first one is titled '*Dynamic Viscoelastic Response of Bone*' and was conducted by R. C. Tennyson, R. Ewert and V. Niranjana. It uses a macroscopic mechanical testing called the Split-Hopkinson-bar technique with beef-femur bone samples.[10]

The second is titled '*A viscoelastic, viscoplastic model of cortical bone valid at low and high strain rates*' and is conducted by T.P.M.Johnson, S.Socrate and M.C.Boyce. This paper combines the datasets

from three preexisting studies (one by J. H. McElhaney using an air gun-type testing machine; one by J. L. Wood using tension testing machines; and one by R. D. Growinshield and M. H. Pope using a Instron TT-CM 1 testing machine and a drop hammer device) which are all macroscopic mechanical testing using either human bones or bovine bones.[16]

Finally the third research study is titled '*Viscoelastic properties of human cortical bone tissue depend on gender and elastic modulus*'. It was conducted by Ziheng Wu, Timothy C. Ovaert and Glen L. Niebur. As opposed to the previous ones, this one consists of nanoindentation testing (therefore at a microscopic scale) with human femur samples.[17]

An interrogation *ad rem* would consider the accuracy when using bovine bone samples to model and predict human bone parameters and behaviours. On this, clear evidences of the validity of bovine-based experimentation results can be found.[18, 19] Fletcher et al. demonstrate the low variability in bone density and pullout force of bovine samples and their customisable potential (demineralisation for bone density modification) making them suitable to human modelisations. It is even shown that juvenile bovine samples have extremely low variability in density and are within the normal range of healthy human adult bone density; while the human samples often used unexpectedly show much larger variability. Bovine bone macroscopic dimensions also make them suitable as a model for biomechanical and fracture fixation testing for human bones.[18] Other species are also used for their similarities. Canine cortical bone best resembled human bone, Aerssens et al. showed. Porcine bone also demonstrated equivalent bone density and fracture stress values to that of human samples.[20] The similarities between human and other species is such that it is even sometimes - in the case of bovine-derived bones for example - used in bone regeneration for some patients.[21] Comparing studies against each other - like in this report - when these use different sample types (either human or animal) therefore remains valid.

2.1 Dynamic Viscoelastic Response of Bone

This paper investigates potential correlation between viscoelastic parameters (namely ε and η) and

post-mortem age of bone samples. The samples were stored in water and refrigerated with the same conditions for up to 38 days. Some 240 days old dried bone samples were also investigated. The testing is at the macroscopic scale and uses the split-Hopkinson pressure bar in compression. This method consists of two bars symmetrically located around the bone sample. A cylindrical projectile collides with the input bar which transmits an elastic-strain pulse to the output bar, while passing through the specimen. Strain gages then measure strain pulses at the input and output. From these values, the viscosity and the elasticity can be computed.

The viscoelastic model provided very good results to determine the material properties (Section 3 of this report). However, no clear correlations between post-mortem age and these properties were found, apart from the fact that age does affect the values of these properties.

2.2 A viscoelastic, viscoplastic model of cortical bone valid at low and high strain rates

This paper proposes a new viscoelastic combined with viscoplastic model for cortical bone, based on three datasets from previously carried-out studies. The researchers demonstrated with their Maxwell-Weichert model that higher-rate viscoelasticity is largely due to hydraulic stiffening with the bone's fluid flow. They identified two viscoelastic regimes, the first due to osteons undergoing shear mechanisms between its collagen fibers and due to shearing motion between lamellar compact matrix layers and at the bone's cement line interfaces.[17]

Regarding the viscoplastic behavior of bone, they found that it was due to sacrificial bonds, when these break and reform at a rate-dependent manner.

2.3 Viscoelastic properties of human cortical bone tissue depend on gender and elastic modulus

Correlations between viscoelasticity of bone and rate-dependent failure due to damage in the bone tissue were investigated in this second paper. The relationship with gender and elastic modulus was also

analyzed. Nanoindentation testing technique was performed on hydrated samples. Nanoindentation is widely used for measuring mechanical properties in natural and synthesised biomaterials at nano or microscropic scale. In this paper 20 indents were performed in 20 different osteons for each sample.

The research demonstrated that the macroscopic viscoelastic behaviour of cortical bone measured with torsion tests seems to have different underlying mechanisms than that of microscopic-scaled viscoelastic behaviours measured with nanoindentation in the osteons. It also showed the role of fluid flow in viscoelastic behaviour and this parameter's relationship with fatigue.

3 Tennyson et al.'s simplified Voigt model

In this paper, Tennyson et al. first considered "a three-element model composed of a spring in series with a parallel arrangement of another spring and dashpot"[10] which actually corresponds to the more complex Zener model (which has also other variations). Zener models are used for reversible deformation of rubber-like materials. Although Zener models provide better qualitative descriptions of both the creep and the stress relaxation, these are quite complex to use compared to Maxwell or Kelvin-Voigt models. The latter is what the researchers decided to use *in fine*, with the following equation.

$$\sigma = E\varepsilon + \eta\dot{\varepsilon} \quad (1)$$

One major aspect which leads to assumptions determining the viscoelastic model, is the linearity. Models are designed with a non-linear or linear characterisation. Nonlinearity typically applies to very large deformations or when the properties of the material change under deformations. Here, cortical bone was considered as a material characterized by a classical linear viscoelastic solid. The researchers estimated that for the range of strain rates used (10 to 450 sec^{-1}) in the split-Hopkinson-bar tests, the degree of nonlinearity they observed in the response curves is small enough to be neglected. This allowed them to use a Kelvin-Voigt model which was simplified from the Zener first-order linear viscoelastic model.

The elastic moduli - determined from the Kelvin-Voigt model - were plotted as a function of strain rate for their samples of varying post-mortem age. The Kelvin-Voigt model plot fitted the data for the 11 days post-mortem age sample very well, demonstrating a suitable choice of viscoelastic model. The researchers determined a long-term elastic constant E of 18 GPa and a viscosity η of 2.1×10^4 Pa-sec for the bovine cortical bone sample.

4 Johnson et al.'s Maxwell—Weichert model

In the second paper by Johnson et al. the researchers investigated both the viscoelastic and viscoplastic behaviours. For this report, we will only consider their work on viscoelasticity. Cortical bone is also characterized linearly, however this time a Maxwell–Weichert model is used. It consists of multiple springs and dashpots in parallel. In this case, we have two Maxwell elements and a single spring in parallel (Figure 2). The single spring describes the ideal elastic response of bone (i.e. equilibrium modulus) while the Maxwell elements correspond to the stress relaxation represented in Equation 2 with stress relaxing exponentially over time. Since the Maxwell model by itself presents some limitations (section 6), the more complex version of the Maxwell-Wiechert was chosen and provided a better modelling. It also exists under even more complex forms with more than two Maxwell branches - which did not seem necessary here.

Each Maxwell units in parallel, represented in the equation, by the last two products, have different parameter values for elasticity and viscosity. The values found from using the Maxwell-Weichert model are detailed in Table 1. When an instantaneous response is considered like in this case, we do not have an isolated dash-pot. A good balance between accuracy and complexity must be determined when designing such a model: the more Maxwell units, the better the accuracy will be - while also increasing the number of material parameters therefore complexifying the computations. As mentioned, here two Maxwell units were enough. In fact, the researchers state that a three-Maxwell-branches model was examined (as well as a non-linear dashpot model) which was found to in-

crease complexity with little improvement in *capturing* the viscoelastic behaviour of cortical bone.[16]

$$\sigma(t) = E_0 \dot{\epsilon} t + \eta_1 \dot{\epsilon} (1 - e^{-\frac{E_1 t}{\eta_1}}) + \eta_2 \dot{\epsilon} (1 - e^{-\frac{E_2 t}{\eta_2}}) \quad (2)$$

Another reasoning behind this choice of model is that there are two distinct strain-rate sensitivity (i.e. viscoelastic) regimes according to the researchers. This means that a simple linear model would not be sufficient to fully and accurately characterize the viscoelastic properties of cortical bone over a larger rate of strain-rate. The first regime is below 300 sec^{-1} strain rate where a linear relationship is observed. The second regime - above 300 sec^{-1} - is also linear but with shorter time constant. Over the thirteen research datasets that were plotted, this double trend was observed.

Data set	E_0 (GPa)	E_1 (GPa)	E_2 (GPa)	η_1 (MPa s)	η_2 (kPa s)
McElhaney	16.2	4.4	23.5	132	227
Wood	9.9	6.3	6.2	128	82
Crowninshield and Pope	9.9	1.9	10.8	115	35

Table 1: Fit model parameters for elasticity and viscosity determined by Johnson et al. from the Maxwell-Weichert model [16]

The researchers assume that the two regimes described previously would mean two viscous mechanisms can be found within the microstructure of the bone. In this case, nanoidentation testing might be required to investigate further. They also demonstrated how viscoelasticity is decreased with decreasing water content in the bone composition. Water and fluids seem to also explain the second linear regime, since experiments on dry bone samples lack this double trend. The other linear regime is due to the intra- and inter-osteonal shearing mechanisms such as mineralized collagen fibers and lamellar layers shearing in a rate-sensitive manner.[16]

Another observation was that collagen fiber viscoelasticity - as it was mentioned in the introduction - is responsible for most of the overall bone viscoelasticity at low strain rates.

5 Ziheng et al.'s Burgers model

In this study Ziheng et al. decided to use the Burgers model combining 4 elements. This model represents

the viscoelastic behaviour of a material with a better precision while minimizing the number of spring and dashpot elements, therefore also minimizing the number of material parameters. These models consist of a combination of a Kelvin-Voigt (spring and dashpot in parallel) and Maxwell element (spring and dashpot in series) in series (Figure 2). In addition to accuracy, this model offers advantages when researchers aim to represent steady state creep, primary creep or instantaneous elastic response. The burgers model seem to show an initial instantaneous deformation when it is under constant load, with a following retarded flow. For stress relaxation, after load removal the instantaneous recovery is again followed by a slow and incomplete recovery of the material sample.

The Burgers model of viscoelasticity for stress relaxation is given by Equation 3 and for creep by Equation 4. These are then modified in the context of this research by Ziheng et al. to Equation 5 to provide a relationship between indentation depth (h) and viscoelastic properties. For this, the stress on the right of the equation is replaced with the applied load and tip angle function ; then the left side of the equation is replaced by the square of indentation depth.

$$\sigma(t) = \frac{\epsilon_0}{A} [(q_1 - q_2/\tau_{\epsilon 1})e^{-t/\tau_{\epsilon 1}} - (q_1 - q_2/\tau_{\epsilon 2})e^{-t/\tau_{\epsilon 2}}] \quad (3)$$

τ is the retardation time or creep time constant depending on the literature and is equal to the viscosity parameter divided by the elasticity parameter. This ratio can also be observed in equation 2 for the Maxwell-Weichert model.

$$\epsilon(t) = \sigma_0 \left[\frac{1}{E_1} + \frac{t}{\eta_1} + \frac{1}{E_2} (1 - e^{-t/\tau_\sigma}) \right] \quad (4)$$

$$h^2(t) = \frac{\pi}{2} P_0 \cot \alpha \left[\frac{1}{E_1} + \frac{1}{E_2} (1 - e^{-\frac{E_2 t}{\eta_2}}) + \frac{1}{\eta_1} t \right] \quad (5)$$

In Equation 5, h(t) is the indentation depth, P is the applied load, α is the equivalent cone semi-angle (70.3°) the E and η components are related to the creep compliance and the viscosity for the Burgers model elements.

The researchers found the coefficient of variation of the creep time constant to average around 18.3 for interstitial tissue and 17.7 for osteonal tissue - which was showed in section 4 to be clearly correlated with the viscoelastic trends of cortical bone.

The validation of the use of Burgers model is demonstrated by the correlation between the researchers' results and those of previous studies also using nanoindentation or other macroscopic testing. Ziheng et al. found in fact that with a Berkovich indenter (i.e. their testing technology), the creep time constant based on Burgers model is between 1 and 5 s - which is very close to the other research papers.[17]

6 Comparison and evaluation of the models

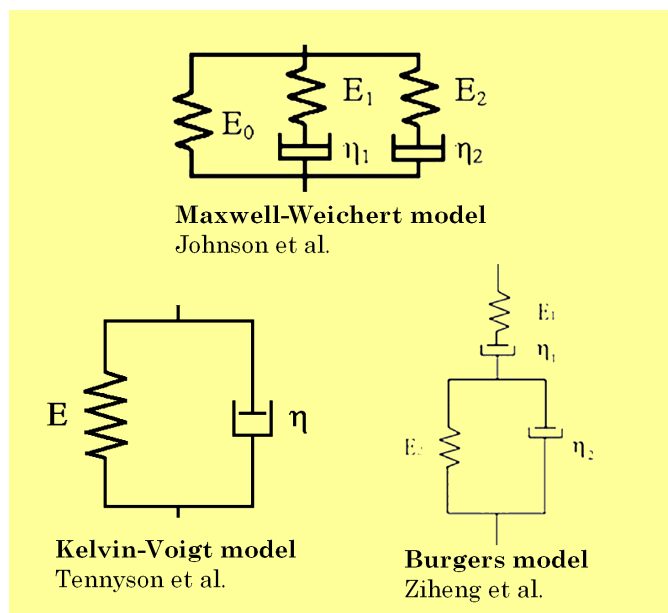


Figure 2: Schematic spring-dashpot representation of the three viscoelastic models analysed. [10, 16, 17]

For the first paper by Tennyson et al., beyond the satisfactory choice of viscoelastic model showed in the previous sections of this report, their technique also demonstrated great results. Its use to determine the viscoelastic properties of cortical bone showed highly reliable results. It is important to note that this research was conducted in the seventies and published in 1972; and still recently this paper is widely cited and considered as an important reference regarding viscoelasticity of bone.

The Kelvin-Voigt model shows limitations regarding relaxation. While it is extremely good and accurate with modelling creep, it is much less accurate for relaxation modelling. The dynamic compressive

loading test carried out by Tennyson et al. here corresponded to creep, since it a time dependent deformation under constant stress. This was in fact one of the researchers' assumptions "Since the stress rate existing during the test remained at $(1 \text{ to } 6) 10^8 \text{ psi/sec}$, it can thus be considered constant." [10] Some limitations can still be pointed out such as the level of simplifications and some assumptions made. Certainly these did not seem to pose any issue in the accuracy of results here, but Kelvin-Voigt was often said to be insufficient to model the viscoelasticity of bone in more recent research of the 2010s[16, 17]. The *compromissum* seems to be that for macroscopic mechanical testing methods, a simple Kelvin-Voigt model is sufficient (based on a significant 82 tests with 43 bones[10]) and under 10 to 450 sec^{-1} strain rates. However for microscopic testing (i.e. nanoindentation) it would be much less ideal then.

The simple Maxwell model shows some limitations when it comes to creep where some inaccuracy can be observed. In this case, a Maxwell-Weichert model is preferred - also because it is better adapted to long time period investigations of strain. Johnson et al. state that they tested other viscoelastic models before choosing the Maxwell-Weichert one. Kelvin-Voigt, simple Maxwell and standard linear solid models were analysed in their preliminary research. The scientists say they all lacked "ability to reliably characterize the increasing tangent modulus of the material over the full range of strain rates employed" [16]. Another argument the researchers made is that their model does more than "simply acting as a curve-fitting tool" [16], it is also said to show the underling nature of the viscoelastic response of bone.

Overall a strength of this Maxwell-Weichert is that it also allowed to put the viscoelastic behaviours of cortical bone into perspective with the microstructural mechanisms of bone related to its composition. Phenomena at various scales are here connected with one another. The model does not however account for the material anisotropy and is not adapted to failure prediction, which is not a major issue given the clear scope of the Johnson et al.'s research.

The Burgers model from Ziheng et al. demonstrated great fitting with regards to the time-dependent creep behaviour which was captured (correlation coefficient R squared is 0.99). Burgers is therefore suitable for nanoindentation testing.

It may seem sensitive to compare these three models contextually-speaking, when these are based on different mechanical testing methods, different scales (macro. vs micro.) and different strain rate ranges. Additionally, these studies do not investigate the exact same correlations related to viscoelasticity. Especially when it comes to the more complex nanoindentation techniques, we may argue that only the Burgers model would suit. We could however carry out the same equation transformations (Equations 3 to 5) for a Maxwell-Weichert or standard linear solid model, making them also suitable for microscopic analysis.

7 Conclusion

Both Maxwell—Weichert and Kelvin-Voigt models described in this report presented satisfactory results and good fit with the experimentation data. In this perspective we could validate both models as suitable for macroscopic scale viscoelastic testing. However more credit may be given to Johnson et al.'s paper and therefore to the Maxwell-Weichert model since it was applied to a wider range of strain rates (0.001

to 1500 sec^{-1} for Johnson et al. against 10 to 450 sec^{-1} for Tennyson et al.) and to data extrated from a larger scale of tests (69 tests for McElhaney, 120 specimens for Wood, 71 specimens for Crowninshield and Pope). Also when considering the complexity of bio-materials in terms of their microstructure and potential anisotropy, improvements in making the model arrangements more complex seem necessary, especially for Tennyson et al.'s paper.

When it comes to microscopic mechanical testing with techniques such as nanoindentation, the Burgers model are very useful and greatly accurate. The model curves showed a very good correlation and demonstrated excellent fits to the data in the research by Ziheng et al. Solid accuracy and rationality for Burgers model is shown here.

With regards to measuring and analyzing the suitability of a viscoelastic model, this report shows that it is essential to carefully consider the context, which includes the correlations investigated, the scale of the test, and the various ranges. The balance between the precision of the model and the complexity of the computations is also to be carefully considered and adapted to the specific research involved.

References

- [1] Toshiya Iyo, Yasuyuki Maki, Naoki Sasaki, and Mitsuo Nakata. Anisotropic viscoelastic properties of cortical bone. *Journal of Biomechanics*, 37(9):1433–1437, 2004.
- [2] Peter Augat and Sandra Schorlemmer. The role of cortical bone and its microstructure in bone strength. *Age and ageing*, 35(suppl_2):ii27–ii31, 2006.
- [3] Susan M Ott. Cortical or trabecular bone: what's the difference? *American journal of nephrology*, 47(6):373–376, 2018.
- [4] Holger Ritzel, Michael Amling, Martin Pösl, Michael Hahn, and Günter Delling. The thickness of human vertebral cortical bone and its changes in aging and osteoporosis: A histomorphometric analysis of the complete spinal column from thirty-seven autopsy specimens. *Journal of Bone and Mineral Research*, 12(1):89–95, 1997.
- [5] Yohann Bala, Roger Zebaze, and Ego Seeman. Role of cortical bone in bone fragility. *Current opinion in rheumatology*, 27(4):406–413, 2015.
- [6] E.M Ooms, J.G.C Wolke, M.T van de Heuvel, B Jeschke, and J.A Jansen. Histological evaluation of the bone response to calcium phosphate cement implanted in cortical bone. *Biomaterials*, 24(6):989–1000, 2003.
- [7] Roderic S Lakes, J Lawrence Katz, and Sanford S Sternstein. Viscoelastic properties of wet cortical bone—i. torsional and biaxial studies. *Journal of Biomechanics*, 12(9):657–678, 1979.

- Page 8